

**DEPARTMENT FOR TRANSPORT 2010**  
**Transport and Works Act 1992**  
**Transport and Works (Applications and Objections Procedure)**  
**(England and Wales) Rules 2006**  
**Chiltern Railways TWA Application 2010 to the Secretary of State**

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**TRAIN – TRACK – SOIL – VIBRATION INTERACTIONS**

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I attach a published research paper<sup>1</sup> that demonstrates that the ground vibration spectra produced by passing railway trains "depend strongly on the mechanical properties of soil on the axle loads of the carriages and on the geometrical parameters of the track and train."

I submit this document in support of my earlier statements that Chiltern Railways and ERM have failed in the following:

- failed to account for any of the effects of complex train-track-coil interaction
- failed to heed the requests from objectors to carry out additional investigations due to the complex nature of local soil and landscape (e.g. the existing elevation on which the track is located near Lakeside)
- failed to carry out measurements at multiple distances from the track

and thus are in no position to make predictions about vibration effects that can be relied on to any reasonable extent.

Therefore, the assessment of consequences (in terms of vibration and noise) of track improvement and any mitigation **must be:**

- performance-based
- involve transparent, independent and quantifiable measurements of noise and vibration
- rely on the comparison of vibration levels before and after the scheme

In addition, I note that if and when these matters appear too technical, the inquiry rules allow a technical specialist to be appointed to assist.

Alexander Korsunsky

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<sup>1</sup> Krylov V, Ferguson C. "Calculation of Low-Frequency Ground Vibrations from Railway Trains", *Applied Acoustics* 42 (1994) 199-213



## Calculation of Low-Frequency Ground Vibrations from Railway Trains

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### ABSTRACT

*Understanding physical mechanisms of railway-induced ground vibrations is an essential prerequisite for reducing their impact on the built environment. In this paper the theory of generation of low-frequency ground vibrations by moving trains in the framework of the quasi-static wheel pressure effect is developed using the Green's function formalism. Mechanical properties of the rail track and parameters of train and soil (including contact nonlinearity of the track–soil system) are taken into account in a theoretical model that considers generation of elastic waves by each sleeper of the track subjected to the action of all wheel axles. The analytical expressions have been derived and the numerical calculations have been carried out for the vibration spectra generated by trains. It is shown that these spectra depend strongly on the mechanical properties of soil, on the axle loads of the carriages and on the geometrical parameters of the track and train. Some practical methods are suggested to modify the spectra of railway-induced ground vibrations and to decrease their level in the chosen frequency intervals.*

### 1 INTRODUCTION

Railway-induced ground vibrations may cause noticeable movements of nearby buildings that result in damage or disamenity, both directly and by generating structure-borne noise.<sup>1–5</sup> Both effects depend strongly on the spectra of railway-induced ground vibrations that, in turn, are deter-

mined by the contributions of different excitation mechanisms and by soil properties.

Spectra of railway-induced ground vibrations, including train-speed-dependent components, have been studied experimentally,<sup>6,7</sup> and qualitative analysis of the quasi-static excitation mechanism has been attempted.<sup>8</sup> However, no rigorous theoretical investigations of railway-generated ground vibrations have been carried out and no quantitatively calculated spectra exist in the literature.

The most important excitation mechanisms of ground vibrations by moving trains are quasi-static pressure from wheel axles onto the track, the effects of joints in unwelded rails, unevenness of wheels or rails, and the effects of carriage- and wheel-axle bending vibrations which occur at their natural frequencies.

In this paper we consider only the first mechanism—the effect of quasi-static pressure of wheel axles onto the track-soil system. In the case of welded rails and perfect wheels, this mechanism is the major contributor to train-speed-dependent components of the low-frequency vibration spectra (up to 50 Hz), including the so-called passage frequency  $f_p = v/d$ , where  $v$  is train speed and  $d$  is distance between sleepers.

In the following sections we state the problem and discuss the role of a track deflection curve in generating ground vibrations. Then we describe generation of ground vibrations by individual sleepers and by periodic sequences of sleepers subject to the action of all wheel axles. In the final section we calculate vibration spectra and discuss some simple and effective methods of reducing vibration levels at selected frequencies.

## 2 STATEMENT OF THE PROBLEM

We consider a train having  $N$  carriages and moving with speed  $v$  on welded track at grade with sleeper periodicity  $d$ . The excitation being considered results from load forces applied to the track from each wheel axle causing downward deflection of the track. For the low-frequency components of the spectra (up to 100 Hz) most relevant for ground vibration, these deflections can be considered as quasi-static, producing a wave-like motion along the track with speed  $v$  and resulting in a distribution of the axle load over all the sleepers involved in the deflection distance  $2x_0$ . The term  $x_0$  is determined as a characteristic distance between the axle load location and the point along the track where deflection is still noticeable. Thus, each sleeper acts as a vertical force applied to the ground during the time necessary for a deflection curve to pass through the sleeper and, according to classical theories, this should result in generation of elastic ground vibrations.

Since, in the relevant frequency band, the characteristic wavelengths of generated elastic waves are much larger than the sleeper dimensions, each sleeper can be considered as a point-source vertical force. The problem then requires superposition of the elastic fields radiated by all sleepers caused by the passage of a single axle, taking into account the time difference between forces and their different locations in space. Then we take into account that each carriage has two bogies, separated by the distance  $M$ , which in turn comprise two axles separated by the distance  $a$ . Finally, we sum the contributions of all  $N$  carriages of the train that evidently has periodicity  $L$ , where  $L$  is the carriage length.

An important aspect of the above is calculation of the track deflection curve as a function of the elastic properties of track and soil and of the magnitude of the axle load. The form of the deflection curve determines the ground vibration frequency spectrum generated by each sleeper. In turn, these spectra strongly affect the total vibration spectrum generated by a passing train.

### 3 DETERMINATION OF THE TRACK DEFLECTION CURVE

Since the track deflection distance is greater than the distance between sleepers, one can ignore the influence of rail periodic support by sleepers in the quasi-static problem of track deflection under the impact of a moving load. Instead we treat a track (i.e. two parallel rails with periodically fastened sleepers) as an Euler–Bernoulli elastic beam of uniform weight  $p$  lying on an elastic or viscoelastic foundation occupying the semispace  $z > 0$ . Deflection of such a beam subjected to a vertical point load has been considered by many authors. The classical solution<sup>9,10</sup> starts with the static beam equation that takes into account a force reaction in the elastic foundation proportional to the deflection magnitude  $w$ :

$$EI \partial^4 w / \partial x^4 + \alpha w = F_m \delta(x) + p \quad (1)$$

Here  $F_m$  is a vertical point force,  $E$  is Young's modulus and  $I$  is the cross-sectional momentum of the beam,  $\alpha$  is the proportionality coefficient of the elastic foundation,  $x$  is the distance along the beam, and  $\delta(x)$  is Dirac's delta-function.

If the force  $F_m$  is applied at  $x = 0$ , the solution for  $w$  has the form

$$w = (F_m / 8EI\beta^3) \exp(-\beta|x|) [\cos(\beta x) + \sin(\beta|x|)] + p/\alpha \quad (2)$$

where  $\beta = (\alpha/4EI)^{1/4}$ . According to eqn (2), one can take  $x_0 = \pi/\beta$  as the total deflection distance.

The constant  $\alpha$  in eqns (1) and (2) depends particularly on the stiffness of the ground and of the rubber pads inserted between rail and sleepers.

Calculation of  $\alpha$  for typical British Rail tracks<sup>11,12</sup> gives the values  $\alpha = 61.8 \text{ MN/m}^2$  and  $\beta = 1.28 \text{ m}^{-1}$ . For the typical distance between sleepers,  $d = 0.7 \text{ m}$ , this implies that about seven sleepers are involved in the deflection curve associated with each axle.

A more recent approach to an analogous problem in mechanics<sup>10</sup> acknowledges that for this type of loading tensile stresses cannot be transmitted between the beam and the elastic foundation. This model is more appropriate for track-soil contacts which can respond only to compressive stresses. The single equation (1) should be replaced by

$$EI \partial^4 w / \partial x^4 + \alpha w = F_m \delta(x) + p \quad \text{for } |x| < x_0, z > 0 \quad (3)$$

$$EI \partial^4 w / \partial x^4 = p \quad \text{for } |x| > x_0, z < 0 \quad (4)$$

where  $x_0$  denotes now the location (unknown) where the beam might separate from the elastic foundation. Thus the contact nonlinearity of a real boundary between track and ground is taken into account.

Analysis of the system of eqns (3) and (4) shows<sup>10</sup> that for values of the axle load  $F_m \leq F_{cr} = (2p/\beta) \exp(\pi)$ , the simple classical solution of eqn (2) which describes a continuous contact between track and foundation remains valid. However, for  $F_m > F_{cr}$  the solution becomes more complicated and involves so-called peripheral bulges of the track in the area  $x_0 < x < x_1$  with loss of contact between track and soil. In this case eqns (3) and (4) are solved numerically for coordinates  $x_0$  and  $x_1$  of the deformed track where it intersects ground level ( $z = 0$ ), and for the five coefficients describing the shape of the deflection curve as a function of applied load.

For our purposes it is sufficient to use a rather rough analytical approximation of the deflection function for  $F_m > F_{cr}$ , where only one parameter  $x_0$  as a function of  $F_m$  is taken into account:

$$\begin{aligned} w &= p/\alpha + (F_m/8EI\beta^3) \cos(\pi x/2x_0) & \text{for } |x| < x_0 \\ w &= 0 & \text{for } |x| > x_0 \end{aligned} \quad (5)$$

This allows us to consider the most important effects of the track deflection function  $w(x)$  on the ground vibration spectra without additional complications. Numerical data for  $x_0$  as a function of  $F_m$  calculated in Ref. 10 over the range  $1.6/\beta < x_0 < \pi/\beta$  can be approximated by the equation

$$x_0 = (1/\beta) \{ \pi - [0.4 \log(\beta F_m / 2e^\pi p)]^{0.3} \} \quad (6)$$

which describes the decrease in  $x_0$  with increasing applied load  $F_m$ . Note that this approximation is invalid for very large loads when  $x_0$  approaches the sleeper period  $d$ . In this case, the effect of periodic sleeper support should be taken into account.<sup>12</sup>

#### 4 GENERATION OF GROUND VIBRATIONS BY INDIVIDUAL SLEEPERS

To calculate ground vibrations generated by individual sleepers let us consider each sleeper as a point-source vertical force applied to the surface  $z = 0$  at  $x = 0$  and  $y = 0$ , with time dependence determined by the passage of the deflection curve through the sleeper:

$$P(t) = F_m [2w(vt)/w_{\max}] (d/x_0) \quad (7)$$

where  $w_{\max}$  is the maximum value of  $w(vt)$ . Terms on the right of  $F_m$  take into account the distribution of axle load between sleepers within the deflection curve.

We shall now make use of results from the well-known axisymmetric Lamb problem for the excitation of an elastic semispace by a vertical point force applied to the surface.<sup>13,14</sup> The solution of this problem describes the corresponding components of the dynamic Green's tensor  $G_{zi}$  (or, for simplicity, the components of the Green's function) for the elastic semispace. This function satisfies the dynamic equations of elasticity for a semispace, assumed isotropic and homogeneous, and the appropriate boundary conditions. The equation of motion takes the form of the Lamé equation:

$$(\lambda + 2\mu) \text{grad div } \mathbf{u} - \mu \text{rot rot } \mathbf{u} - \rho_0 \partial^2 \mathbf{u} / \partial t^2 = 0 \quad (8)$$

where  $\mathbf{u}$  is the vector of particle displacements,  $\lambda$  and  $\mu$  are the elastic Lamé constants, and  $\rho_0$  is the ground mass density. On the boundary  $z = 0$  the displacement field  $\mathbf{u}$  should satisfy the boundary conditions of absence of normal elastic stresses everywhere, apart from the point  $x = 0$ ,  $y = 0$  of force application where an integrable singularity would appear. This implies that on the boundary  $z = 0$  the following boundary conditions should hold:

$$\sigma_{zz} = \lambda u_{nn} + 2\mu u_{zz} = -P(t)\delta(x)\delta(y) \quad (9)$$

$$\sigma_{zl} = 2\mu u_{zl} = 0 \quad (10)$$

where  $\sigma_{ij}$  are the components of the stress tensor,  $u_{ij} = 1/2(\partial u_i / \partial x_j + \partial u_j / \partial x_i)$  are the components of the linearised deformation tensor (here  $u_i$  are the components of the displacement vector  $\mathbf{u}$ );  $i$  and  $j$  take the values  $x, y, z$ ;  $l$  takes the values  $x$  and  $y$ ; and double subscripts imply summation as usual. In addition to obeying eqns (8)–(10), the generated elastic waves should radiate outwards from the point  $x = 0, y = 0$  (the Sommerfeld radiation condition).

The solution of such problems is usually obtained in terms of a Fourier integral representation over both time and space.<sup>13,15</sup> Calculation

of this integral in the plane of complex wavenumber  $k$  represents the generated elastic field (for distances beyond several wavelengths) as a superposition of Rayleigh surface waves and bulk longitudinal and shear elastic waves propagating along the surface with strong geometrical attenuation. In what follows only Rayleigh surface waves are considered because they transfer most of the vibrational energy. For these waves the spectral density of the vertical vibrational velocity at  $z = 0$  may be written in the form

$$v_z(\rho, \omega) = P(\omega)G_{zz}(\rho, \omega) = V(\omega)(1/\sqrt{\rho}) \exp(ik\rho) \exp(-i3\pi/4) \quad (11)$$

where

$$V(\omega) = (\pi/2)^{1/2}P(\omega)(-i\omega)q(k_R)^{1/2}k_t^2/\mu F'(k_R) \quad (12)$$

Here  $\rho = [(x - x')^2 + (y - y')^2]^{1/2}$  is the distance between the source (with current coordinates  $x', y'$ ) and the point of observation (with coordinates  $x, y$ ),  $\omega = 2\pi f$  is a circular frequency,  $k_R = \omega/c_R$  is the wavenumber of a Rayleigh surface wave where  $c_R$  is the Rayleigh wave propagation velocity,  $k_l = \omega/c_l$  and  $k_t = \omega/c_t$  are the wavenumbers of longitudinal and shear bulk elastic waves, where  $c_l = [(\lambda + 2\mu)/\rho_0]^{1/2}$  and  $c_t = (\mu/\rho_0)^{1/2}$  are longitudinal and shear propagation velocities,  $\lambda$  and  $\mu$  are Lamé constants,  $\rho_0$  is the ground mass density, and  $q = (k_R^2 - k_t^2)^{1/2}$ . The factor  $F'(k_R)$  is a derivative of the Rayleigh determinant

$$F(k) = (2k^2 - k_t^2)^2 - 4k^2(k^2 - k_t^2)^{1/2}(k^2 - k_l^2)^{1/2} \quad (13)$$

taken for  $k = k_R$ , and

$$P(\omega) = (1/2\pi) \int_{-\infty}^{\infty} P(t) \exp(i\omega t) dt \quad (14)$$

is a Fourier transform of  $P(t)$ . The factor  $1/\sqrt{\rho}$  in eqn (11) describes the cylindrical spreading of Rayleigh waves with propagation distance. Note that setting  $F(k) = 0$  gives the value of  $k_R$  and hence the Rayleigh wave velocity as a function of the elastic constants and mass density.

For a viscoelastic semispace the elastic constants  $\lambda$  and  $\mu$  should be considered as complex numbers taking into account frequency-dependent attenuation and dispersion of elastic waves.<sup>16</sup> This would make  $c_R$ ,  $c_l$  and  $c_t$  complex also, resulting in a decrease in the amplitudes of all waves with distance and a broadening of their time-forms because of velocity dispersion. In the calculations below we will not take velocity dispersion into account since it is rather small for the homogeneous semispace being considered.

It is seen from eqns (11) and (12) that the Fourier transform  $P(\omega)$  plays a very important role in determining the spectra of radiated waves. In the case under consideration,  $P(\omega)$  should be determined separately for  $F_m \leq F_{cr}$  and for  $F_m > F_{cr}$  for which the deflection function  $w(x)$  is

described by eqns (2) and (5) respectively. Substituting these expressions into eqn (7) and then into eqn (14), one can easily obtain the corresponding analytical formulae for the Fourier transform  $P(\omega)$ :

$$P(\omega) = (F_m d / \pi x_0) \{ (2\beta v + \omega) / [(\beta v)^2 + (\beta v + \omega)^2] + (2\beta v - \omega) / [(\beta v)^2 + (\beta v - \omega)^2] \}, \quad \text{for } F_m \leq F_{cr} \quad (15)$$

$$P(\omega) = (F_m d / \pi x_0) (2\pi v / x_0) \cos(\omega x_0 / v) / [\omega^2 - (\pi v / 2x_0)^2], \quad \text{for } F_m > F_{cr} \quad (16)$$

where  $x_0$  is determined by eqn (6).

One can generalise these results to describe the action of two axle loads separated by the distance  $a$  (the case of a bogie). The time dependence of the bogie force  $P_b(t)$  may then be represented in the form

$$P_b(t) = P(t + a/2v) + P(t - a/2v) \quad (17)$$

Substituting eqn (17) into eqn (14) instead of  $P(t)$  gives the result

$$P_b(\omega) = 2P(\omega) \cos(\omega a / 2v) \quad (18)$$

## 5 CONSIDERATION OF ALL SLEEPERS AND AXLES

To calculate the vibration field radiated by a complete moving train requires the superposition of fields generated by each sleeper activated by all axles of all carriages, with the time and space differences between sources (sleepers) being taken into account.

Using the Green's function formalism this may be written in the form

$$v_z(x, y, \omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P(x', y', \omega) G_{zz}(\rho, \omega) dx' dy' \quad (19)$$

where  $P(x', y', \omega)$  describes the total distribution of forces along the track. This distribution is found by taking a Fourier transform of the time- and space-dependent track deflection function.

It is useful firstly to consider a single axle load moving with speed  $v$  along the track lying on perfectly elastic ground. Then the load force which makes a wave-like motion along the track may be written in the form

$$P(t, x', y' = 0) = \sum_{m=-\infty}^{\infty} P(t - x'/v) \delta(x' - md) \delta(y') \quad (20)$$

with the Fourier transform

$$P(x', y', \omega) = (1/2\pi) \int_{-\infty}^{\infty} \sum_{m=-\infty}^{\infty} P(t - x'/v) \exp(i\omega t) \delta(x' - md) \delta(y') dt \quad (21)$$

where  $m$  denotes the current number of a sleeper, and  $\delta(x'-md)$  takes the periodic distribution of sleepers into account. Integration yields

$$P(x', y', \omega) = P(\omega) \exp(i\omega x'/v) \sum_{n=-\infty}^{\infty} \delta(x' - md) \delta(y') \quad (22)$$

Substituting eqn (22) into eqn (19) and using the properties of integrating delta-functions we have, after taking eqns (11)–(13) into account, the following expression for the vertical vibration velocity of Rayleigh waves generated at  $x = 0$ ,  $y = y_0$  by a single axle load moving along the track with speed  $v$ :

$$v_z(x = 0, y = y_0, \omega) = V(\omega) \sum_{m=-\infty}^{\infty} \exp[i(\omega/v)md + i(\omega/c_R)\rho_m] / \sqrt{\rho_m} \quad (23)$$

where  $\rho_m = [y_0^2 + (md)^2]^{1/2}$ . Formula (23) shows that a single moving load generates a quasi-discrete spectrum with frequency peaks close to  $f_p s$ , where  $f_p = v/d$  is the so-called passage frequency, and  $s = 1, 2, 3, \dots$ . Appearance of higher-order harmonics of  $f_p$  is caused by the fact that wave pulses radiated by each individual sleeper are determined by the shape of the deflection curve and are generally not sine-like. Deviation from perfect discreteness results from the  $i(\omega/c_R)\rho_m$  term in eqn (23) which takes into account path-length differences of waves propagated from each sleeper to the point of observation.

To take account of all axles and carriages one needs a more complicated load function:

$$P(t, x', y' = 0) = \sum_{m=-\infty}^{\infty} \sum_{n=0}^{N-1} A_n [P(t - (x' + nL)/v) + P(t - (x' + M + nL)/v)] \delta(x' - md) \delta(y') \quad (24)$$

Here  $N$  is the number of carriages,  $M$  is the distance between bogies in each carriage and  $L$  is the total carriage length. Dimensionless quantity  $A_n$  is an amplitude weight-factor to account for different carriage masses, but for simplicity we will suppose all carriage masses to be equal ( $A_n = 1$ ).

Substituting eqn (24) into eqn (21) and then into eqn (19), and making simple transformations similar to the above, one obtains the following expression for the frequency spectra of vertical vibrations at  $z = 0$  generated by a moving train:

$$v_z(x = 0, y = y_0, \omega) = V(\omega) \sum_{m=-\infty}^{\infty} \sum_{n=0}^{N-1} [\exp(-\gamma\omega\rho_m/c_R) / \sqrt{\rho_m}] \times [1 + \exp(iM\omega/v)] \exp(i(\omega/v)(md + nL) + i(\omega/c_R)\rho_m) \quad (25)$$

In writing eqn (25) we account for attenuation in soil by replacing  $1/c_R$  in the exponentials by the complex value  $1/c_R + i\gamma/c_R$ , where  $\gamma \ll 1$  is a constant describing the 'strength' of dissipation of Rayleigh waves in soil (eqn (25) implies a linear frequency dependence of soil attenuation, in agreement with experimental data).<sup>17,18</sup> We recall that  $V(\omega)$  is determined by eqn (12) in which the spectrum  $P(\omega)$  is defined by eqns (15), (16) and (18). Note that the factor  $1/\sqrt{\rho_m}$  in eqn (25) takes into account the cylindrical spreading of Rayleigh waves generated by each sleeper.

The summation over  $m$  in eqn (25) considers an infinite number of sleepers. However, the contribution of remote sleepers is small because of soil attenuation and cylindrical spreading, and a few hundred sleepers are adequate for practical calculations.

It follows from eqn (25) that the spectrum of train-induced vibrations is quasi-discrete, with the maxima at frequencies determined by the condition  $(\omega/v)(md + nL) = 2\pi l$ , where  $l = 1, 2, 3, \dots$ . Obviously,  $n = 0$  corresponds to the passage frequencies  $f_{ps}$  determined by the sleeper period  $d$ . Other more frequent maxima are determined either by the carriage length  $L$  ( $m = 0$ ) or by a combination of both parameters (for  $n \neq 0, m \neq 0$ ).

There are many zeros present in the train vibration spectra. These zeros may be used in practice for suppressing vibrations at chosen frequencies. The most important zeros are those which do not depend on a number of sleepers or carriages and are determined only by the geometrical parameters of a carriage. One of these zeros is determined by the distance  $a$  between the wheel axles in a bogie (see eqn (18) for the spectrum  $P_b$ ). Setting  $P_b$  to zero, one can obtain  $f_z = (v/a)(n + 1/2)$  for zero-frequencies. If, for instance, we want to use this condition to suppress one of the train passage frequencies  $f_{ps}$ , we should choose  $f_z$  to be equal to  $f_{ps}$ . It follows from this that the value of  $a$  should be determined by

$$a = (d/s)(n + 1/2) \quad (26)$$

It is sensible to choose a value of  $a$  close to existing values. For British Rail heavy-freight carriages  $a = 2.2$  m usually. Therefore, to suppress the main passage frequency ( $s = 1$ ) one can choose  $a = 2.45$  m corresponding to  $n = 3$  in eqn (26).

Other important zero frequencies reflect the distance  $M$  between bogies in a carriage (see the terms  $1 + \exp(iM\omega/v)$  in eqn (25)). Condition (26) is also valid for this case if  $a$  in eqn (26) is replaced by  $M$ . The value of  $M$  providing suppression of the main passage frequency which is closest to the British Rail standard ( $M = 4.88$  m) is 4.55 m, corresponding to  $n = 6$ .

## 6 NUMERICAL CALCULATIONS AND DISCUSSION

Numerical calculations of train-induced ground vibrations described by eqns (25), (12), (15), (16), (18) and (6) have been carried out for different values of applied load  $F_m$ , train speed  $v$ , and soil attenuation coefficient  $\gamma$ , and for different geometrical parameters of both track and train:  $d$ ,  $a$ ,  $L$ ,  $M$ . Summation over  $m$  in eqn (25) was carried out from  $m = -Q/2$  to  $m = Q/2$ . The value of  $Q$  is chosen such that the corresponding length of track  $Qd$  is sufficiently greater than the total train length  $NL$  and the attenuation distance of Rayleigh waves at the frequency band considered. For instance, if the number of carriages  $N$  is equal to 10 and  $L = 8.3$  m, then for typical values of  $d = 0.7$  m and  $\gamma = 0.005$  the number of sleepers  $Q$  for  $f > 10$  Hz should be at least 200. In our calculations we use  $Q = 300$ . The elastic parameters of the soil considered were  $c_R = 250$  m/s,  $c_t = 272$  m/s,  $c_l = 471$  m/s (corresponding to a Poisson's ratio of  $\sigma = 0.25$ ). The mass density of soil  $\rho_0$  was set at  $2000 \text{ kg/m}^3$ .

Figure 1 shows the theoretical frequency spectra for a five-carriage train moving with speed  $v = 50$  km/h. Calculations have been carried out for an axle load of 100 kN (below the critical value  $F_{cr} = 108.3$  kN determined for a typical track weight with concrete sleepers,  $p = 3$  kN/m) and for a load of 200 kN (above the critical value). Geometrical parameters of the train were chosen typical of British Rail heavy-freight trains:<sup>6</sup>  $L = 8.3$  m,  $M = 4.88$  m and  $a = 2.2$  m. The soil attenuation parameter  $\gamma$  was chosen to be 0.004 78, observation distance  $y_0$  was 30 m. According to the figure, both spectra have maxima at the train passage frequencies (the main passage frequency for  $v = 50$  km/h is 20 Hz) and at frequencies determined by train geometrical parameters. Generation is more efficient at 200 kN axle load, especially at higher frequencies; this results mainly from the sharper form of the deflection curve  $w(t - x/v)$  and hence the wider spectrum  $P(\omega)$ . Note that the shapes and intensities (in dB) of the calculated spectra are in agreement with existing experimental data.<sup>6,7</sup>

The vibration spectra calculated for different train speeds,  $v = 50$  km/h and  $v = 40$  km/h, are shown in Fig. 2 for an axle load of 100 kN. The parameters of track, train and soil, as well as the observation distance  $y_0$ , are the same as before. As expected, changing the train speed displaces the spectral maxima.

Figure 3 illustrates the influence of observation distance  $y_0$  on the behaviour of the vibration spectra for a train speed of 50 km/h and an axle load of 200 kN. The soil attenuation coefficient has been set equal to  $\gamma = 0.004 78$ . In agreement with the linear frequency dependence of attenuation, the higher-frequency components are more strongly damped than the lower ones.

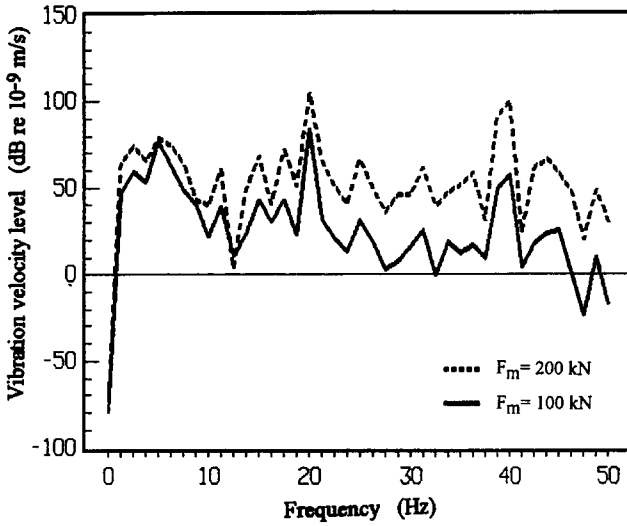


Fig. 1. Ground vibration spectra for different axle loads,  $F_m$ .

The effect of soil attenuation  $\gamma$  on the vibration spectra is shown in Fig. 4 for the fixed observation distance  $y_0 = 30$  m, all other conditions being the same as in Fig. 3. It is seen that with increasing  $\gamma$  all spectral components are damped, the damping being stronger at higher frequencies. Note that spectral components at the passage frequencies are less affected by soil attenuation than those at combined frequencies determined by train geometrical parameters. This is because the passage

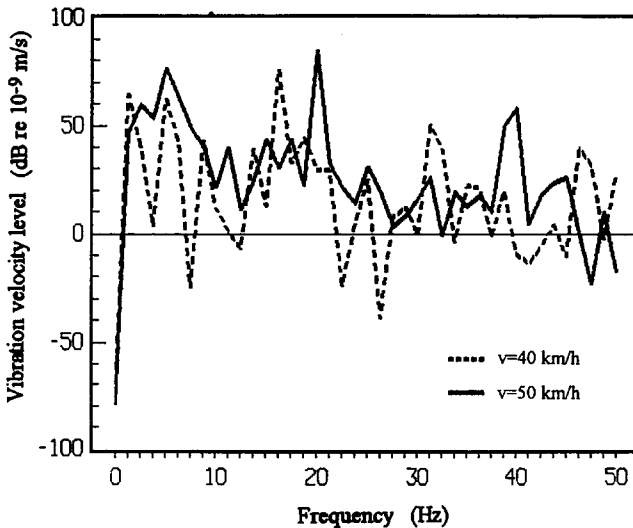


Fig. 2. Influence of train speed  $v$  on ground vibration spectra.

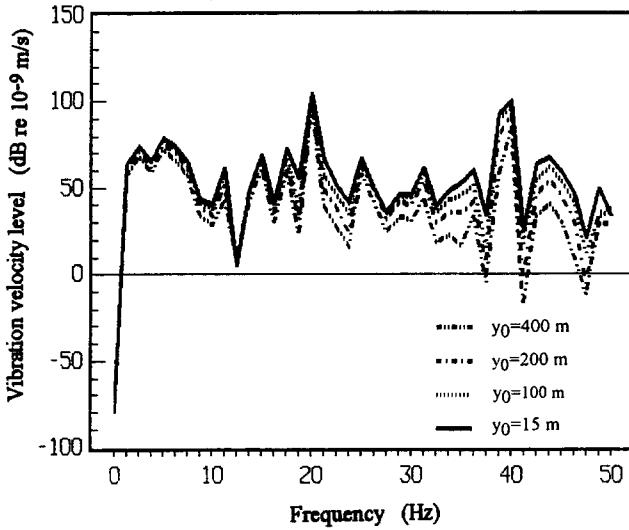


Fig. 3. Effect of observation distance  $y_0$  on ground vibration spectra.

frequency response is determined mainly by sleepers close to the point of observation, whereas the response at combined frequencies is determined by signals from remote carriages which are strongly attenuated.

Figure 5 shows how choice of the distance  $a$  between axles in a bogie can suppress ground vibration intensity at the main passage frequency  $f_p = v/d$ . The corresponding distance  $a = 2.45$  m was calculated using

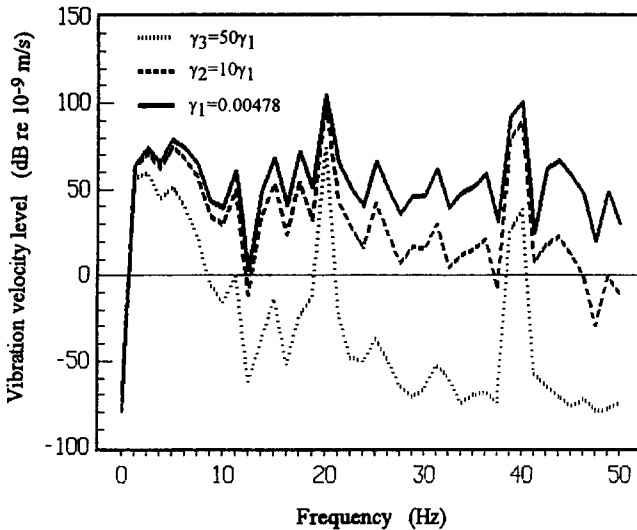
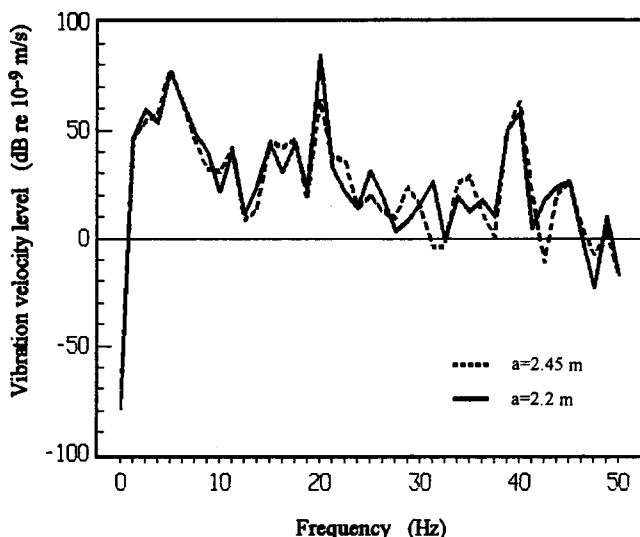


Fig. 4. Ground vibration spectra for different values of soil attenuation coefficient,  $\gamma$ .



**Fig. 5.** Suppression of ground vibrations at the main passage frequency by selecting the appropriate distance  $a$  between the axles in a bogie.

eqn (26) for  $d = 0.7$  m. The axle load was 100 kN, the train speed  $v = 50$  km/h, and the distance  $y_0 = 30$  m, other parameters being the same as in previous figures. According to the figure, the vibration level at the lowest passage frequency is suppressed by about 20 dB, i.e. a factor of 10 relative to that for  $a = 2.2$  m. Of course, the same effect could be achieved by changing the sleeper period  $d$ .

Figure 6 shows how changing the distance  $M$  between bogies in the carriage from 4.88 m to 4.55 m, calculated from eqn (26) also reduces vibration intensity at the passage frequency. Other parameters are the same as in the previous figure. Again, the suppression at the lowest passage frequency is about 20 dB.

## 7 CONCLUSIONS

Generation of ground vibrations by moving trains has been considered theoretically using the Green's function formalism. Expressions for the ground vibration spectra have been obtained as functions of track, train and soil parameters.

Numerical calculations have been carried out for different physical and geometric properties of track, train and soil. The shapes and intensities of the spectra are in good agreement with experimental data. This suggests that the excitation mechanisms have been correctly identified.

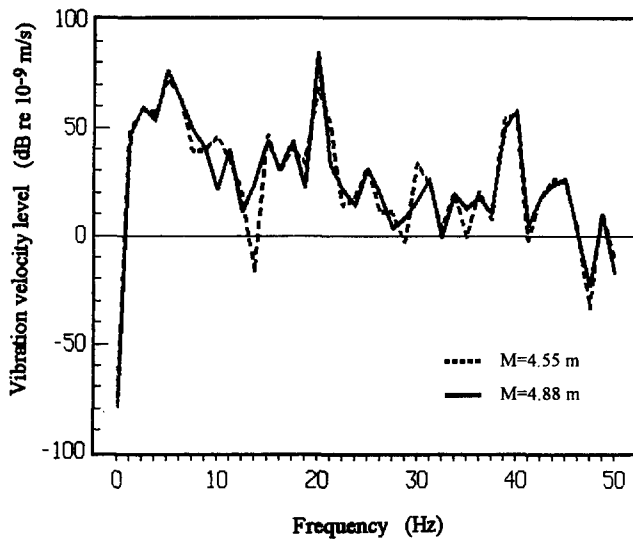


Fig. 6. Suppression of ground vibrations at the main passage frequency by selecting the appropriate distance  $M$  between bogies in a carriage.

Vibration spectra depend strongly on the axle loads of the carriages: if the axle load exceeds a critical value beyond which peripheral bulges appear in the track, the vibration level increases significantly, especially at higher frequencies.

By proper selection of the distance between wheel axles in a bogie, and between bogies in a carriage (or between sleepers in a track), it is possible effectively to suppress vibration levels at the train passage frequencies.

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